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5 REACTANCE 3 TERMINAL PEAKING CIRCUIT

by

Leo E. Foley

Research Report No. PIBMRI-1119-63

for

The Air Force Office of Scientific Research
The U.S. Army Research Office
The Office of Naval Research

Contract No. AF-AFOSR-62-295

February 28, 1963

MRI

POLYTECHNIC INSTITUTE OF BROOKLYN
MICROWAVE RESEARCH INSTITUTE

ELECTRICAL ENGINEERING DEPARTMENT

Report Number PIBMRI 1119-(

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Abstract

The high frequency response of the inter-stage coupling network of any amplifier is limited by the total parasitic capacitance that exists between the output terminals of one stage and the following input terminals. By the addition of suitably chosen reactive elements the amplitude, phase, and time responses of the network will be improved. A five reactive network with three terminals is analyzed on a normalized basis and compared to the simple RC network which will be called the uncompensated reference. Two capacitive elements in the network represent the division of the distributed parasitic capacitance into lumped elements while the remaining three elements are physical entities. Values for the reactive elements will be selected for three different criteria: 1) critically damped transient response, 2) maximally flat amplitude response, and 3) linear phase response. After the parameters have been selected the normalized equations for amplitude, time delay, and step response will be derived and plotted for comparative purposes. Pole-zero plots are also drawn to give a more succinct picture of the networks analyzed. The calculations for this report were made with either an IBM 650 computer or a desk calculator. Six significant figures were maintained in the calculations. However, all figures in this report will be rounded off to four places.

Table of Contents

I. TRANSFER IMPEDANCE OF THE NETWORK

a. Introduction

The high frequency representation of the interstage coupling network to be analyzed is shown in figure 1. A constant-current drive is represented by the current I_1 while V_2 is the voltage into the next stage. Therefore, the quantity of interest is the ratio of V_2 to I_1. C_1 and C_2 represent lumped model representations for the parasitic capacitance and L_1, L_2, and C_3 are the peaking elements to be added. R is the load impedance seen at DC.

b. Normalization

By dividing all resistance and inductance by the factor R and multiplying all capacitance by the same factor the network may be impedance scaled such that at zero frequency the load is one ohm. Frequency scaling may be accomplished by dividing all inductance and capacitance by $R(C_1 + C_2)$ such that in the uncompensated case the half-power frequency occurs at ω equal to one radian per second. These operations yield the network shown in figure 2. If $C_1 + C_2 = C$, the parasitic capacitance in the uncompensated case, the following normalization equations will be obtained:

$$q = \frac{C_1}{C} \qquad 1-q = \frac{C_2}{C}$$

$$k_1 = \frac{L_1}{R^2 C} \qquad k_2 = \frac{L_2}{R^2 C}$$

$$q_1 = \frac{C_3}{C} \qquad \phi = \omega RC$$

$$H_{tr} = \frac{Z_{tr}}{R} \qquad p = sRC$$

$$\tau = \frac{t}{RC}$$

By solving the above set the following **denormalizing** equations are obtained:

$$C_1 = qC \qquad C_3 = q_1 C$$

$$C_2 = (1-q)C \qquad Z_{tr} = R\,H_{tr}$$

$$L_1 = k_1 R^2 C \qquad L_2 = k_2 R^2 C$$

$$t = RC\,\tau \qquad \omega = \frac{\phi}{RC}$$

$$s = \frac{p}{RC}$$

c. Derivation

The ladder reduction method (1) can be used to obtain the normalized transfer impedance of the network shown in figure 3.

$$1 \qquad V_3$$

$$pq_1 + 1 \qquad I_3$$

$$p^2k_2q_1 + pk_2 + 1 \qquad V_2$$

$$p^3(1-q)k_2q_1 + p^2k_2(1-q) + p(1-q) + pq_1 + 1 \qquad I_2$$

$$p^4k_1k_2q_1(1-q) + p^3k_1k_2(1-q) + p^2(1-q)k_1 + p^2k_1q_1 + p^2k_2q_1 + pk_1 + pk_2 + 1 \qquad V_1$$

$$p^5k_1k_2qq_1(1-q) + p^4k_1k_2q(1-q) + p^3\left[k_1q(1-q) + q_1qk_1 + k_2qq_1 + k_2q_1(1-q)\right] + p^2\left[qk_1 + qk_2 + k_2(1-q)\right] + p\left[q + 1 - q + q_1\right] + 1 \qquad I_1$$

$$H_{tr} = \frac{p^2k_2q_1 + pk_2 + 1}{p^5k_1k_2qq_1(1-q) + p^4k_1k_2q(1-q) + p^3\left[k_1q(1-q) + q_1(qk_1 + k_2)\right] + p^2\left[qk_1 + k_2\right] + p\left[1 + q_1\right] + 1}$$

The following substitutions can be made:

$$k_1 q = a \qquad 1-q = b$$

$$k_2 = c \qquad q_1 = d$$

$$Htr = \frac{p^2 cd + pc + 1}{p^5 abcd + p^4 abc + p^3(ab + ad + cd) + p^2(a + c) + p(1 + d) + 1}$$

II. CRITICALLY DAMPED TRANSIENT RESPONSE

a. Selection of Parameters

This criterion requires that all the poles of Htr coalesce at one point on the negative real axis. The Laplace inverse of this pole configuration will be void of any sine or cosine terms and thereby of a critically damped nature. D(p) must be of the form:

$$\left(1+\frac{p}{\alpha}\right)^5 = 1+\frac{5p}{\alpha}+\frac{10p^2}{\alpha^2}+\frac{10p^3}{\alpha^3}+\frac{5p^4}{\alpha^4}+\frac{p^5}{\alpha^5}$$

$$D(p) = 1+(1+d)p+(a+c)p^2+(ab+ad+cd)p^3+abcp^4+abcdp^5$$

By equating the coefficients of equal powers of p the following system of equations are obtained:

$$1+d = \frac{5}{\alpha} \qquad \frac{10}{\alpha^2} = a+c \qquad \frac{10}{\alpha^3} = ab+ad+cd$$

$$\frac{5}{\alpha^4} = abc \qquad \frac{1}{\alpha^5} = abcd$$

This set is solved directly to yield:

$$a = \frac{175}{576} \qquad b = \frac{5}{21} \qquad c = \frac{25}{192}$$

$$d = \frac{1}{24} \qquad \alpha = \frac{24}{5}$$

Solved in terms of the network parameters the following values are obtained and indicated on the network in figure 4 in terms of the denormalizing quantities:

$$q_1 = \frac{1}{24} \qquad q = \frac{16}{21} \qquad 1-q = \frac{5}{21}$$

$$k_2 = \frac{25}{192} \qquad k_1 = \frac{1225}{3072}$$

b. Step Response

$$Htr(p) = \frac{p^2 \frac{25}{4608} + p\frac{25}{192} + 1}{\left(1 + p\frac{5}{24}\right)^5}$$

$$= \left(\frac{24}{5}\right)^5 \frac{p^2\frac{75}{(24)^3} + p\frac{75}{(24)^2} + 1}{\left(p + \frac{24}{5}\right)^5}$$

$$= \frac{K_1}{\left(p + \frac{24}{5}\right)^5} + \frac{K_2}{\left(p + \frac{24}{5}\right)^4} + \frac{K_3}{\left(p + \frac{24}{5}\right)^3} + \frac{K_4}{\left(p + \frac{24}{5}\right)^2} + \frac{K_5}{p + \frac{24}{5}}$$

$$= \frac{\frac{1}{2}\left(\frac{24}{5}\right)^5}{\left(p + \frac{24}{5}\right)^5} + \frac{\frac{3}{8}\left(\frac{24}{5}\right)^4}{\left(p + \frac{24}{5}\right)^4} + \frac{\frac{1}{8}\left(\frac{24}{5}\right)^3}{\left(p + \frac{24}{5}\right)^3}$$

The inverse Laplace of the above yields the impulse response:

$$h(\tau) = \frac{1}{2}\left(\frac{24}{5}\right)^5\frac{1}{4!}\tau^4\exp(-\frac{24}{5}\tau) + \frac{3}{8}\left(\frac{24}{5}\right)^4\frac{1}{3!}\tau^3\exp(-\frac{24}{5}\tau)$$
$$+ \frac{1}{8}\left(\frac{24}{5}\right)^3\frac{1}{2!}\tau^2\exp(-\frac{24}{5}\tau)$$

The step response can be obtained from the above:

$$a(\tau) = \int_0^\tau h(x)\,dx$$

$$a(\tau) = 1 - \exp(-4.8\tau) - 4.8\tau\exp(-4.8\tau) - 11.52\tau^2\exp(-4.8\tau)$$
$$- 16.13\tau^3\exp(-4.8\tau) - 11.06\tau^4\exp(-4.8\tau)$$

The above equation is shown in figure 7 where it is the curve labeled T.

c. Amplitude Response

$$Htr(j\phi) = \frac{1 - \frac{25}{4608}\phi^2 + j\phi\frac{25}{192}}{\left(1 + j\phi\frac{5}{24}\right)^5}$$

$$|Htr|^2 = \frac{\left(1 - \frac{25}{4608}\phi^2\right)^2 + \left(\frac{25}{192}\right)^2\phi^2}{\left(1 + \left(\frac{5}{24}\right)^5\phi^2\right)^5}$$

$$= \frac{1 + .006104\,\phi^2 + .2943\text{X}10^{-4}\,\phi^4}{1 + .2170\,\phi^2 + .01884\,\phi^4 + .8176\text{X}10^{-3}\,\phi^6 + 17.74\text{X}10^{-6}\,\phi^8 + .1540\text{X}10^{-6}\,\phi^{10}}$$

The $|Htr|$ is shown in figure 8 as the T curve.

d. Normalized Time Delay

$$D = \frac{\theta\,(\phi)}{\theta\,(\phi)\Big|_{\lim \phi \to 0}}$$

$$-\theta\,(\phi) = \tan^{-1}\frac{NrDi - NiDr}{NrDr + NiDi}$$

where Nr, Dr, Ni, and Di are the real and imaginary parts of the numerator and the denominator of $Htr(j\phi)$.

$$Htr(j\phi) = \frac{1 - .005425\,\phi^2 + .1302j\phi}{1 - .4340\,\phi^2 + .009419\,\phi^4 + j\,(1.042\,\phi + .3925\text{X}10^{-3}\,\phi^5 - .09042\,\phi^3)}$$

$$-\theta\,(\phi) = \tan^{-1}\frac{.9115\,\phi - .03956\phi^3 - .3434\text{X}10^{-3}\phi^5 - 2.129\text{X}10^{-6}\,\phi^7}{1 - .3038\,\phi^2}$$

$$D = \frac{-\theta\,(\phi)}{.9115\,\phi}$$

The above equation is shown in figure 9 as the T curve.

III. MAXIMALLY FLAT AMPLITUDE RESPONSE

a. Selection of Parameters

This criterion requires the maximum number of derivatives of the $|Htr|$ versus ϕ vanish in sequence at ϕ equal to zero. An equivalent condition, which will be used below, requires that the maximum number of derivatives of $|Htr|^2$ versus ϕ^2 vanish in sequence at ϕ equal to zero. This condition may be derived as follows:

$$Htr(p) = \frac{1 + a_1 p + a_2 p^2 + a_3 p^3 + \dots}{1 + b_1 p + b_2 p^2 + b_3 p^3 + \dots}$$

$$Htr(j\phi) = \frac{1 + ja_1 \phi - a_2 \phi^2 - ja_3 \phi^3 + \dots}{1 + jb_1 \phi - b_2 \phi^2 - jb_3 \phi^3 + \dots}$$

$$|Htr|^2 = \frac{(1 - a_2\phi^2 + \dots)^2 + (a_1\phi - a_3\phi^3 + \dots)^2}{(1 - b_2\phi^2 + \dots)^2 + (b_1\phi - b_3\phi^3 + \dots)^2}$$

$$= \frac{1 + m_2\phi^2 + m_4\phi^4 + \dots}{1 + n_2\phi^2 + n_4\phi^4 + \dots}$$

$$= \frac{N(\phi^2)}{D(\phi^2)}$$

If $D(\phi^2)$ is divided into $N(\phi^2)$ the following ascending series in ϕ^2 is obtained:

$$|Htr|^2 = 1 + (m_2 - n_2)\phi^2 + \left[(m_4 - n_4) - n_2(m_2 - n_2)\right]\phi^4 + \dots$$

From the above expression it can be seen that if the corresponding coefficients of the $|Htr|^2$ are set equal to each other ($m_2 = n_2$, $m_4 = n_4$, etc.) the conditions for maximally flat amplitude will be satisfied.

For the network considered:

$$|Htr|^2 = \frac{1+\phi^2(c^2-2cd)+\phi^4c^2d^2}{1+\phi^2\left[(1+d)^2-2(a+c)\right]+\phi^4\left[(a+c)^2+2abc-2(ab+ad+cd)(1+d)\right]+\phi^6\left[(ab+ad+cd)^2+2abcd(1+d)-2abc(a+c)\right]+\phi^8\left[a^2b^2c^2-2abcd(ab+ad+cd)\right]+\phi^{10}a^2b^2c^2d^2}$$

Equating corresponding terms gives the following system of equations:

$$c^2-2cd = (1+d)^2-2(a+c)$$

$$c^2d^2 = (a+c)^2+2abc-2(ab+ad+cd)(1+d)$$

$$0 = (ab+ad+cd)^2+2abcd(1+d)-2abc(a+c)$$

$$0 = a^2b^2c^2-2abcd(ab+ad+cd)$$

Using the Newton-Raphson method on the above system yielded the following results from an IBM 650 computer:

$$a = .3002$$

$$b = .4504$$

$$c = .2931$$

$$d = .1014$$

Solved in terms of the network parameters the following values are obtained and indicated on the circuit shown in figure 5 in terms of the denormalizing quantities:

$$q = .5496 \qquad k_2 = .2931$$

$$k_1 = .5462 \qquad q_1 = .1014$$

$$1-q = .4504$$

b. Amplitude Response

$$Htr(j\phi) = \frac{1+j\phi.2932-\phi^2.02973}{1+j\phi\,1.101-\phi^2.5933-j\phi^3.1954+\phi^4.03963+j\phi^5.004018}$$

$$|Htr|^2 = \frac{1+.02647\phi^2+.0008841\phi^4}{1+.02647\phi^2+.0008841\phi^4+16.17\text{X}10^{-6}\phi^{10}}$$

The $|Htr|$ is shown on figure 8 as the A curve.

c. Normalized Time Delay

$$-\theta(\phi) = \frac{.8083\phi - .05420\phi^3 - .001790\phi^5 - .0001195\phi^7}{1 - .3002\phi^2}$$

$$D = \frac{-\theta(\phi)}{.8083\phi}$$

The above equation is shown on figure 9 as the A curve.

d. Step Response

$$Htr(p) = 7.396 \frac{p^2 + 9.862p + 33.65}{p^5 + 9.862p^4 + 48.62p^3 + 147.6p^2 + 274.1p + 248.9}$$

$$= 7.396 \frac{(p+4.931-3.055j)(p+4.931+3.055j)}{(p+2.963)(p+2.443-1.7j)(p+2.443+1.7j)(p+1.007+2.91j)(p+1.007-2.91j)}$$

$$A(p) = \frac{Htr(p)}{p}$$

$$= \frac{K_0}{p} + \frac{K_1}{p+1.007-2.91j} + \frac{K_1^*}{p+1.007+2.91j} + \frac{K_2}{p+2.443-1.7j} + \frac{K_2^*}{p+2.443+1.7j} + \frac{K_3}{p+2.963}$$

$K_0 = 1$ $\quad$ $K_2 = -.3261+.6071j$

$K_1 = .2503+.2643j$ $\quad$ $K_3 = -.8483$

The inverse transform is obtained by using:

$$\mathcal{L}^{-1}\left[\frac{K}{p+\alpha-j\beta} + \frac{K^*}{p+\alpha+j\beta}\right] = 2Re(K)\exp(-\alpha t)\cos\beta t - 2Im(K)\exp(-\alpha t)\sin\beta t$$

$$a(\tau) = 1 - .8483\exp(-2.963\tau) + .5005\exp(-1.007\tau)\cos 2.910\tau - .5287\exp(-1.007\tau)\sin 2.910\tau - .6523\exp(-2.443\tau)\cos 1.7\tau - 1.214\exp(-2.443\tau)\sin 1.7\tau$$

The above equation is shown in figure 7 as the A curve.

IV. LINEAR PHASE RESPONSE (MAXIMALLY FLAT TIME DELAY)

a. Selection of Parameters

The parameters for this criterion are derived by forcing the phase to be linear over as large a range of frequencies as possible. If the phase is initially set equal to some constant times frequency it will possess a Maclaurin series. The corresponding coefficients of this series can be set equal to the phase function of the network. If this procedure is followed in sequence until all degrees of freedom are exhausted it will provide a system of equations that can be solved for the desired parameters as follows:

$$-\theta = \tan^{-1} f(\phi)$$

$$\tan(-\theta) = f(\phi)$$

Let $-\theta = \gamma\phi$, then:

$$\tan\gamma\phi = \gamma\phi + \frac{\gamma^3\phi^3}{3} + \frac{2\gamma^5\phi^5}{15} + \frac{17\gamma^7\phi^7}{315} + \frac{62\gamma^9\phi^9}{2835} + \ldots$$

If $f(\phi) = \gamma_1\phi + \gamma_3\phi^3 + \gamma_5\phi^5 + \gamma_7\phi^7 + \gamma_9\phi^9 + \ldots$ the criterion is satisfied for four network parameters, since γ is an added variable, when:

$$\gamma = \gamma_1 \qquad \frac{\gamma^3}{3} = \gamma_3$$

$$\frac{2\gamma^5}{15} = \gamma_5 \qquad \frac{17\gamma^7}{315} = \gamma_7$$

$$\frac{62\gamma^9}{2835} = \gamma_9$$

For the network considered:

$$-\theta = \tan^{-1} \frac{\phi(1+d-c)+\phi^3(ac+c^2-2cd-cd^2-ab-ad)+\phi^5(2abcd+acd^2+c^2d^2-abc^2)-\phi^7abc^2d^2}{1-a\phi^2}$$

$f(\phi)$ can be divided out to give the following ascending power series:

$$f(\phi) = (1+d-c)\phi + (a+c^2-2cd-cd^2-ab)\phi^3 + (a^2+ac^2-2acd-a^2b+2abcd+c^2d^2-abc^2)\phi^5 + (a^3+a^2c^2-2a^2cd-a^3b+2a^2bcd+ac^2d^2-a^2bc^2-abc^2d^2)\phi^7 + a(a^3+a^2c^2-2a^2cd-a^3b+2a^2bcd+ac^2d^2-a^2bc^2-abc^2d^2)\phi^9 + \ldots$$

The system of equations to be solved is therefore:

$$\gamma = 1+d-c$$

$$\frac{\gamma^3}{3} = a+c^2-2cd-cd^2-ab$$

$$\frac{2\gamma^5}{15} = a^2+ac^2-2acd-a^2b+2abcd+c^2d^2-abc^2$$

$$\frac{17\gamma^7}{315} = a^3+a^2c^2-2a^2cd-a^3b+2a^2bcd+ac^2d^2-a^2bc^2-abc^2d^2$$

$$\frac{62\gamma^9}{2835} = a^4+a^3c^2-2a^3cd-a^4b+2a^3bcd+a^2c^2d^2-a^3bc^2-a^2bc^2d^2$$

The above system solved by the Newton-Raphson method on an IBM 650 computer yielded:

$a = .2987$ $\quad d = .07581$

$b = .3406$ $\quad \gamma = .8565$

$c = .2193$

Solving for the network values gives the following which are also indicated in figure 6 in terms of the denormalizing quantities:

$q = .6594$ $\quad k_2 = .2193$

$k_1 = .4530$ $\quad q_1 = .07581$

$1-q = .3406$

b. Normalized Time Delay

$$-\theta = \tan^{-1} \frac{.8565\phi - .04530\phi^{3} - .0008572\phi^{5} - 28.12\times10^{-6}\phi^{7}}{1 - .2987\phi^{2}}$$

$$D = \frac{-\theta}{.8565\phi}$$

The above equation is shown on figure 9 as the D curve.

c. Step Response

$$Htr(p) = 9.830 \frac{p^{2} + 13.19p + 60.15}{p^{5} + 13.19p^{4} + 83.37p^{3} + 306.3p^{2} + 636.0p + 591.2}$$

$$= 9.830 \frac{(p+6.595+4.080j)(p+6.595-4.080j)}{(p+3.418)(p+2.870-1.601j)(p+2.870+1.601j)(p+2.016+3.457j)(p+2.016-3.457j)}$$

$$A(p) = \frac{Htr(p)}{p}$$

$$= \frac{K_0}{p} + \frac{K_1}{p+2.870-1.601j} + \frac{K_1^*}{p+2.870+1.601j} + \frac{K_2}{p+2.016-3.457j} + \frac{K_2^*}{p+2.016+3.457j} + \frac{K_3}{p+3.418}$$

$K_0 = 1$ $\qquad$ $K_2 = .3612-.08289j$

$K_1 = .1039+1.598j$ $\qquad$ $K_3 = -1.930$

The inverse transform yields the following:

$$a(\tau) = 1-1.930\exp(-3.418\tau)+.2078\exp(-2.870\tau)\cos 1.601\tau -3.196\exp(-2.870\tau)\sin 1.601\tau+.7224\exp(-2.016\tau)\cos 3.457\tau +.1658\exp(-2.016\tau)\sin 3.457\tau$$

The above equation is shown on figure 7 where it is the curve labeled D.

d. Amplitude Response

$$Htr(j\phi) = \frac{1+j\phi .2193-\phi^{2} .01663}{j\phi^{5} .001691+\phi^{4} .02231-j\phi^{3} .1410-\phi^{2} .5180+j\phi 1.076+1}$$

$$|Htr|^2 = \frac{1+.01484\phi^2+.0002764\phi^4}{1+.1214\phi^2+.009553\phi^4+.0004080\phi^6+.00002079\phi^8+2.861X10^{-6}\phi^{10}}$$

The |Htr| is shown on figure 8 as the D curve.

V. RC REFERENCE

The RC reference is obtained by removing all peaking elements. This forces L_1, L_2, and C_3 of figure 1 to zero.

a. Step Response

$$Htr(p) = \frac{1}{1 + p}$$

$$A(p) = \frac{1}{p}\,\frac{1}{1 + p} = \frac{K_1}{p} + \frac{K_2}{1 + p}$$

$$K_1 = 1 \qquad K_2 = -1$$

$$a(\tau) = 1 - \exp(-\tau)$$

b. Amplitude Response

$$|Htr|^2 = \frac{1}{1 + \phi^2}$$

c. Normalized Time Delay

$$D = \frac{\tan^{-1}\phi}{\phi}$$

$a(\tau)$, $|Htr|$, and D are shown as the U curve on figures 7, 8, and 9, respectively.

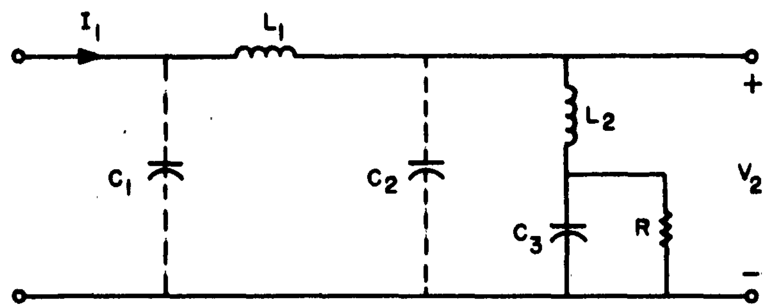


FIG. 1. 5 REACTANCE 3 TERMINAL PEAKING CIRCUIT

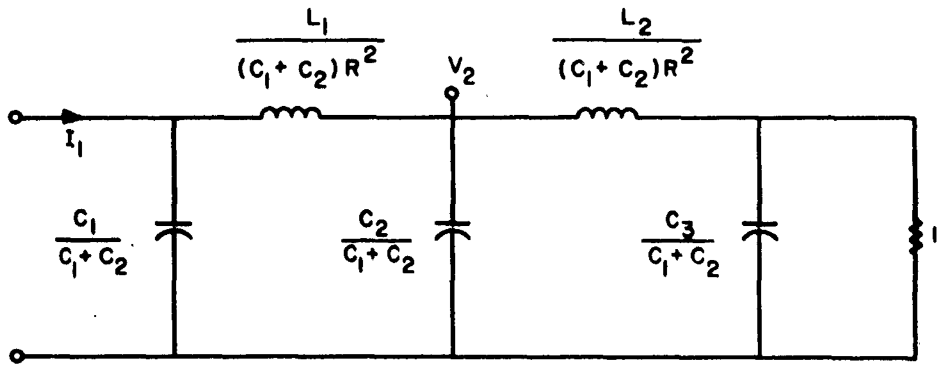


FIG. 2. RESULT OF NORMALIZATION

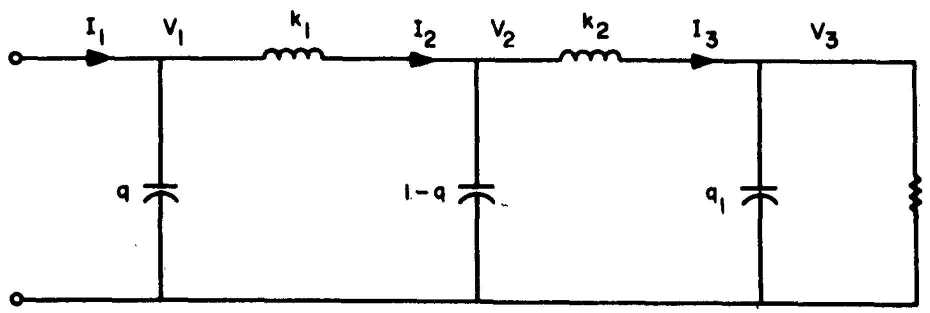


FIG. 3. NORMALIZED NETWORK

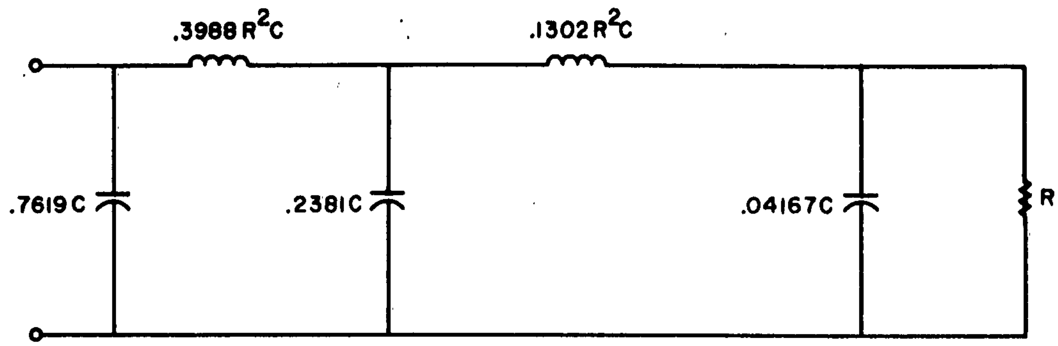


FIG. 4. CRITICALLY DAMPED NETWORK

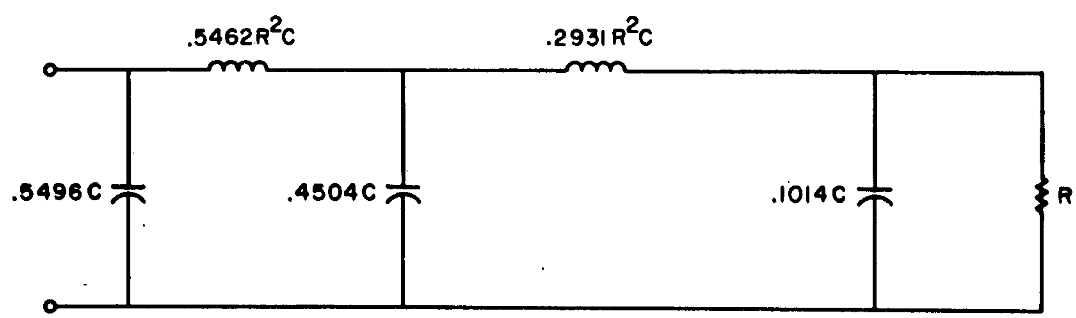


FIG. 5. MAXIMALLY FLAT AMPLITUDE NETWORK

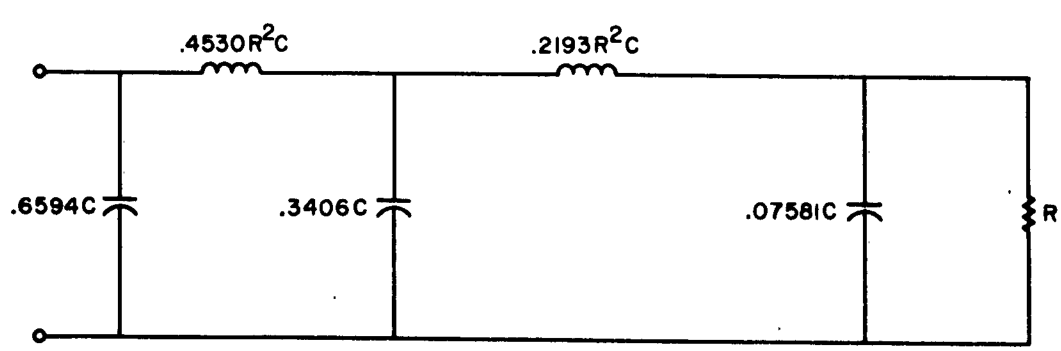


FIG. 6. MAXIMALLY FLAT TIME DELAY NETWORK

FIG. 7 STEP RESPONSE

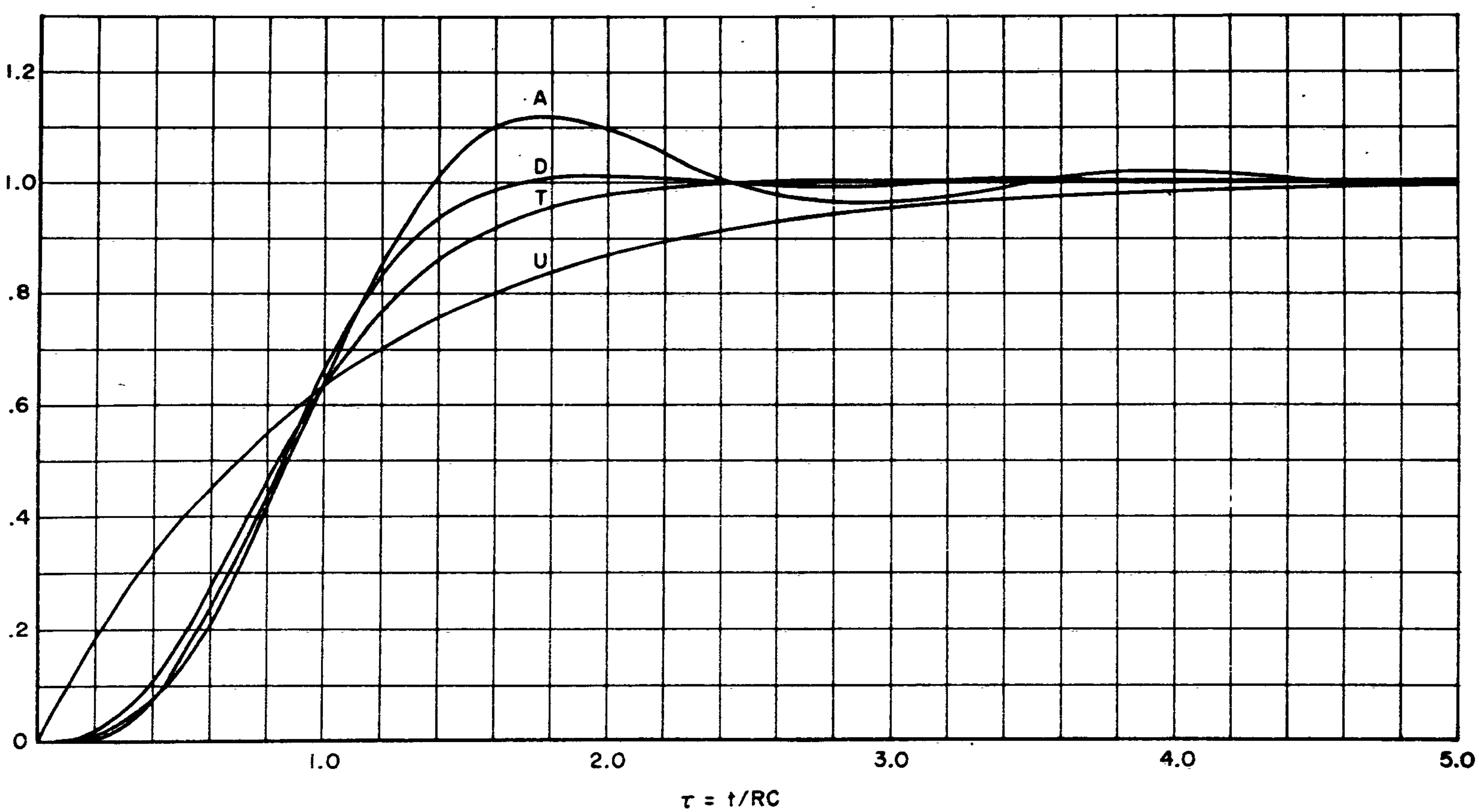
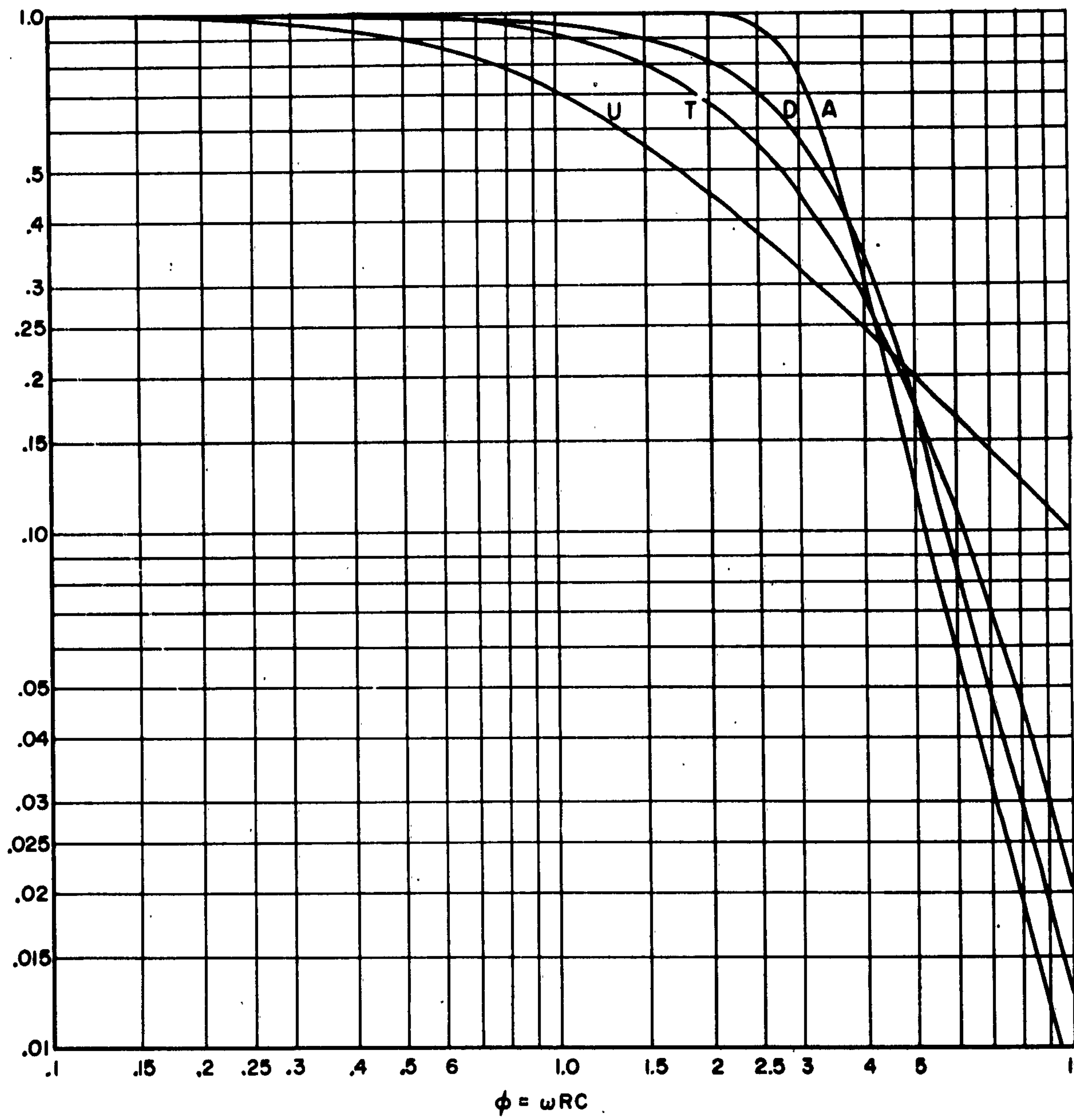


FIG. 8. AMPLITUDE RESPONSE



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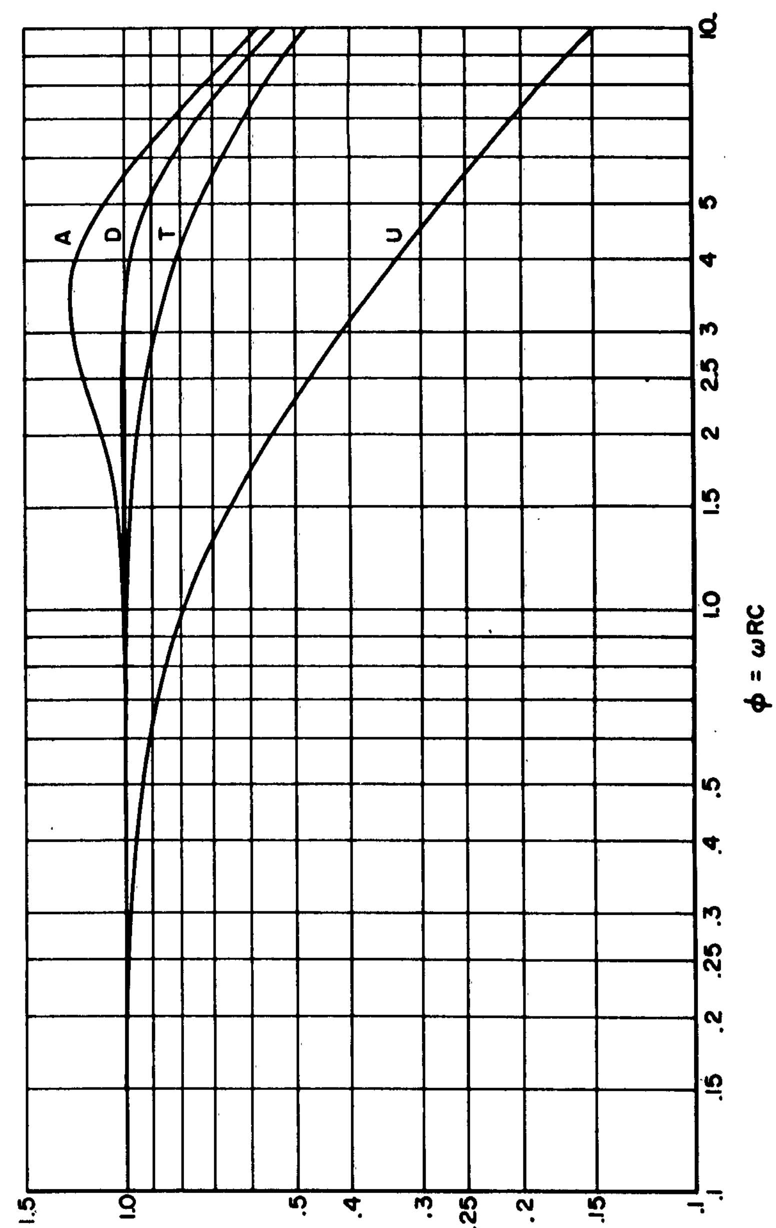
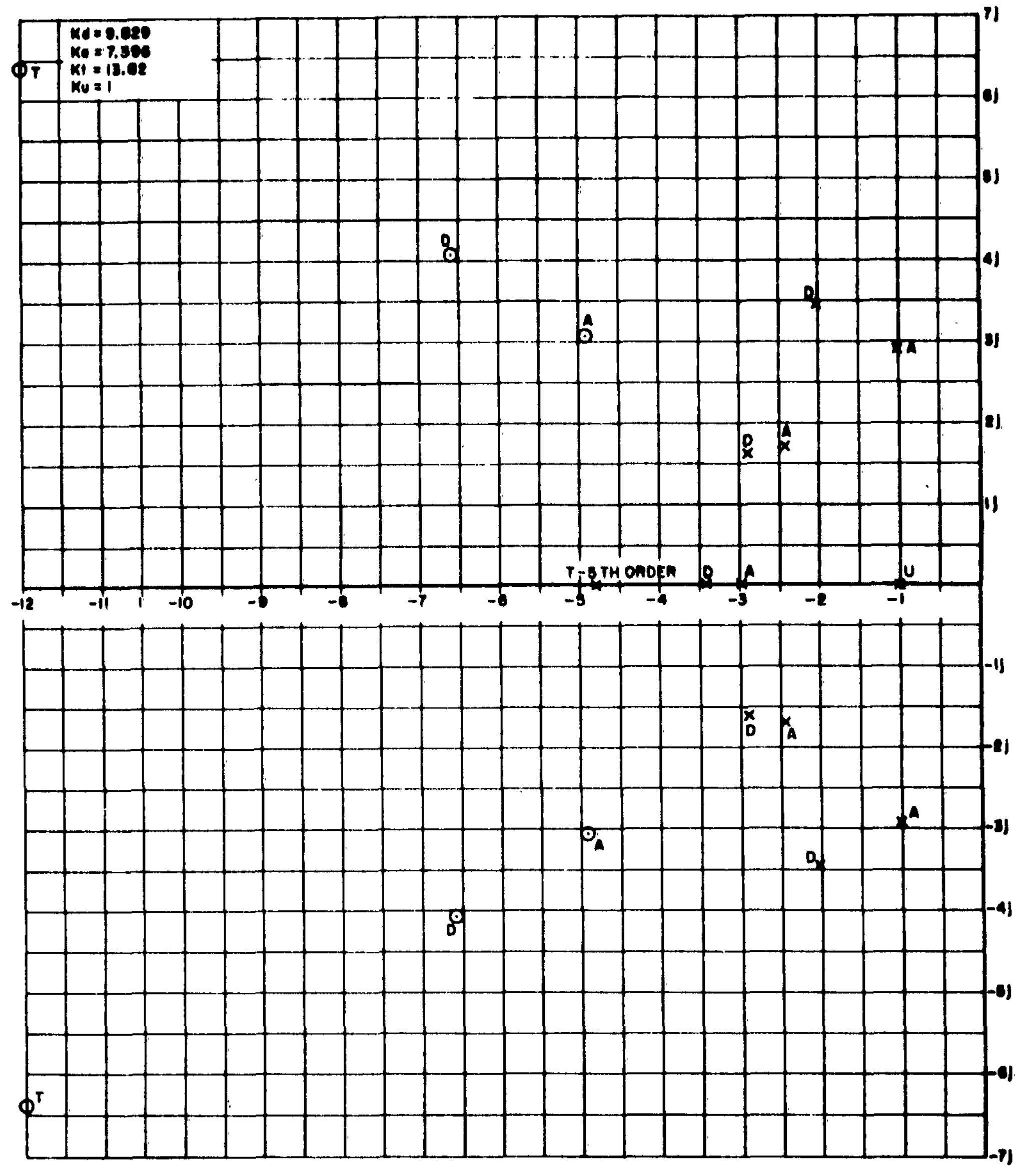


FIG. 9. NORMALIZED TIME DELAY

FIG. 10. POLE-ZERO PLOTS



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VII. TABULATIONS (2), (3), (4), and (5)

a. Step Response

τ	T curve	A curve	D curve	U curve
.1	.0026	.001267	.001651	.0952
.2	.01692			.1813
.3	.05186	.03115	.04000	.2592
.4	.1089			.3297
.5	.1847	.1289	.1572	.3935
.6	.2734			.4512
.7	.3681	.3011	.3443	.5034
.8	.4627			.5507
.9	.5523	.5216		.5934
1.0	.6337	.6364	.6570	.6321
1.1	.7053			.6671
1.2	.7663	.8475	.8321	.6988
1.3	.8172			.7275
1.4	.8588	1.005	.9322	.7534
1.5	.8921			.7768
1.6	.9184	1.094	.9888	.7981
1.8	.9541	1.118		.8347
2.0	.9754	1.096	1.010	.8647
2.2		1.052	1.005	.8892
2.5	.9957	.9889	.9988	.9179
2.7		.9664		.9328
3.0		.9652	.9991	.9502
3.5		1.000	1.001	.9698
4.0		1.013	1.000	.9817
4.3		1.007		.9864
4.6		.9995		.9899
5.0		.9955		.9933

b. Amplitude Response

ϕ	T curve	A curve	D curve	U curve
.2	.9958	1.000	.9979	.9755
.5	.9742	1.000	.9867	.8907
.8	.9356	1.000	.9657	.7809
1.0	.9020	.9999	.9472	.7071
1.5	.7977	.9995	.8832	.5547
2.0	.6785	.9927	.8094	.4472
2.5	.5595	.9413	.6942	.3713
2.7		.8879		.3473
3.0	.4509	.7605	.5773	.3277
3.2		.6545		.2983
3.5		.4962		.2747
4.0	.2813	.2978	.3383	.2425
5.0	.1724	.1176	.1716	.1961
8.0	.04432	.01907	.02921	.1240
10.0	.02094	.008789	.01296	.09950

c. Normalized Time Delay

ϕ	T curve	A curve	D curve	U curve
.5	.9962	1.005	1.000	.9273
1.0	.9840	1.016	1.000	.7853
1.5	.9649	1.040	1.000	.6552
2.0	.9401	1.082	1.000	.5536
2.5		1.152	1.000	.4761
3.0	.8789	1.232	1.000	.4164
4.0		1.224	.9788	.3314
5.0	.7423	1.079	.9131	.2747
7.0	.6200	.8158	.7390	.2041
10.0	.4828	.5805	.5419	.1471

VIII. CONCLUSIONS

From a close scrutiny of the plots many conclusions can be derived. Most analytic figures of merit however are more difficult to evaluate. In most cases they can be obtained by only a cut and try procedure. Therefore, unless a more accurate solution was desired or easily evaluated, the following numerical values will be obtained by graphical interpretation.

Two common figures of merit for a step response are rise time which is the time required to go from the ten percent to the ninety percent point and the amount of overshoot. These quantities are tabulated below:

	A	D	T	U
Rise Time	.81	.87	1.14	2.2
% Overshoot	12	1	0	0

If a slight overshoot can be tolerated, case D(the linear phase criterion) actually will give the best step response.

The three db. or half-power frequency for the various networks are tabulated below:

	A	D	T	U
Ø	3.103(calculated)	2.45	1.86	1

The maximally flat amplitude network will allow an increase in the gain-bandwidth product of the network by a factor of 3.103. Actually in an amplifier composed of several peaked stages the bandwidth increase over the simple RC coupled case will be even greater than 3.103 since the A curve is quite flat before breaking off sharply at approximately the half-power frequency. The figure of 3.103 compares quite favorably with the infinite peaking circuit (6) which has a three db. point at $\phi = 4.02$, and is a 14.5% improvement over a four reactance three terminal network (7) which has a three db. point at $\phi = 2.71$.

A figure of merit for the normalized time delay is the $\pi/4$ point which in the uncompensated network corresponds to the 45 degrees phase point. This point is tabulated below:

	A	D	T	U
ϕ	7.3	6.453 (calculated)	4.4	1

This point does not portray the entire picture since the A curve has considerable overshoot. The $\pi/4$ points for the infinite and four reactance maximally flat time delay networks occur at $\phi = 7.4$ and $\phi = 5.5$, respectively.

The pole-zero plots tend to tie together the three previous plots. It is seen that the poles of the A case are closest to the j axis thereby giving the best frequency response yet the poorest step response. The T case is just the opposite while the D case is a good compromise.

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